Available online at www.sciencedirect.com
SCIENCE DIRECT®
ELSEVIER
Journal of Sound and Vibration 290 (2006) 548–554
JOURNAL OF SOUND AND VIBRATION
www.elsevier.com/locate/jsvi

Discussion

Authors' Reply

A. Zinoviev*, D.A. Bies

School of Mechanical Engineering, The University of Adelaide, North Terrace, Adelaide 5005, Australia

Available online 27 October 2005

In this response to the Discussion by F. Farassat and M.K. Myers we show that these authors did not disprove our criticism concerning the theory derived by Curle–Ffowcs Williams and Hawkings.

1. Introduction

It is the view of the present authors that Farassat and Myers [1] have not successfully refuted the analysis of the theory by Curle [2] and Ffowcs Williams and Hawkings (FW–H) [3] which was presented previously [4,5]. We avoid responding to all comments by Farassat and Myers, which are often not directly related to our previous work. Instead, we consider only the major points which Farassat and Myers appear to be making. For better clarity, the headings by Farassat and Myers are kept intact wherever possible.

2. The assumptions, applications and the current state of the acoustic analogy (AA)

Farassat and Myers stated that, in our previous publications [4,5], we claim that "AA does not give the correct results for some problems". We would like to affirm that by no means do we put into doubt the AA formulation by Lighthill [6], where it is applied to an *unbounded* flow. Our criticism is directed towards the extension of AA to a flow with solid boundaries as done by Curle [2].

Curle's equation should undoubtedly be applicable to sound scattering problems. Indeed, Lighthill's wave equation is derived on the basis of the general momentum and mass conservation laws. Additional assumptions for the region of noise generation, specified by Farassat and Myers, originated from some aspects of Lighthill's derivation. Therefore, regardless of anyone's intentions, Curle's formula should describe the process of sound scattering, where all three assumptions are satisfied. As Curle puts it on page 510 of his paper [2], "… in the present work all such effects as reflexion and diffraction at the solid boundaries are completely accounted for by incorporation into the applied dipole field P_i".

It is the opinion of the present authors that verification of Curle's formula for simple scattering problems is a reliable method to confirm or disprove the formula. If a problem of sound scattering or generation satisfies Lighthill's criteria, Curle's methodology should produce the correct prediction for the scattered/radiated sound. If it does not, one may ask why should this formula be trusted for a case of a complex flow with viscosity, nonlinearity, and vorticity, if it fails in a much simpler case where all these characteristics are absent?

*Corresponding author.
E-mail address: alexei.zinoviev@mecheng.adelaide.edu.au (A. Zinoviev).

0022-460X/$ - see front matter © 2005 Elsevier Ltd. All rights reserved.
doi:10.1016/j.jsv.2005.07.047

In our previous papers [4,5] on the subject, as well as in this response, we showed that Curle's formula, in fact, fails to produce the correct answer in several simple cases, all of which satisfy Lighthill's criteria.

3. On the derivation of Curle's formula

We start the discussion with the theoretical argument since it presents clear evidence against the validity of Curle's equation. We avoid here the detailed analysis of the derivation of Curle's equation, as it can be found in Zinoviev and Bies [4,5].

We would like to draw the attention of the readers to the following point. If Eq. (20) of Farassat and Myers is integrated over a large volume, as they have suggested, it will coincide with our Eqs. (5), (6) of Ref. [4]. Therefore, Farassat and Myers' claim that we did not take into account the retarded time variable is incorrect. Despite not showing the retarded time notation in some of our formulae, we utilize this variable wherever it is necessary.

It is regrettable that Farassat and Myers did not notice the relevant discussion in our previous response [5], where in Section 4 we have provided a clear proof that the derivation of Curle's equation contains a significant error. Due to the importance of this issue, this proof is repeated briefly here.

We begin by integrating Farassat and Myers' equation (20) in variable **y** over a large volume, V, which, in general, should include all the fluid. However, since the turbulent flow occupies only a finite region of space, any finite volume containing all regions in the fluid with non-zero Lighthill's stress tensor, T_{ij}, may be considered as the volume V. It is important that the presence of the rigid boundary within V does not affect this consideration, as the interior of the rigid object can be considered to be filled with a fluid and, therefore, included into the volume V. The resulting equation for such a volume may be written as follows:

$$\iiint_V \left[\frac{\partial^2 T_{ij}}{\partial y_i \partial y_j}\right] \frac{\mathrm{d}\mathbf{v}}{r} = \frac{\partial^2}{\partial x_i \partial x_j} \iiint_V \frac{[T_{ij}]}{r}\,\mathrm{d}\mathbf{y} + \iiint_V \frac{\partial}{\partial y_i}\left(\frac{1}{r}\left[\frac{\partial T_{ij}}{\partial y_j}\right]\right)\mathrm{d}\mathbf{y} - \frac{\partial}{\partial x_i}\iiint_V \frac{\partial}{\partial y_j}\left(\frac{[T_{ij}]}{r}\right). \tag{1}$$

Since the volume V is finite, it has the boundary, Σ, which encloses V. Therefore, the volume integrals over V may be replaced with the surface integrals over Σ by means of the divergence theorem with respect to the variable **y**. This step leads to the following equation:

$$\iiint_V \left[\frac{\partial^2 T_{ij}}{\partial y_i \partial y_j}\right] \frac{\mathrm{d}\mathbf{v}}{r} = \frac{\partial^2}{\partial x_i \partial x_j} \iiint_V \frac{[T_{ij}]}{r}\,\mathrm{d}\mathbf{y} + \iint_\Sigma l_i \left[\frac{\partial T_{ij}}{\partial y_j}\right] \frac{\mathrm{d}S(\mathbf{v})}{r} - \frac{\partial}{\partial x_i}\iint_\Sigma l_j [T_{ij}] \frac{\mathrm{d}S(\mathbf{v})}{r}, \tag{2}$$

where $\mathrm{I} = (l_1, l_2, l_3)$ is the outward normal to Σ.

As the volume V can be arbitrary as long as it contains all regions with turbulent flow, without loss of generality it can be assumed that $T_{ij} = 0$ on Σ. For that reason, one can easily conclude that the surface integrals vanish and Eq. (2) is reduced to

$$\iiint_V \left[\frac{\partial^2 T_{ij}}{\partial y_i \partial y_j}\right] \frac{\mathrm{d}\mathbf{y}}{r} = \frac{\partial^2}{\partial x_i \partial x_y} \iiint_V \frac{[T_{ij}]}{r}\,\mathrm{d}\mathbf{y}. \tag{3}$$

In our previous publications [4,5] on the issue, we considered this derivation in detail and showed that, for Curle's equation to be valid, Eq. (3) should include integrals over the rigid surface, S, so that

$$\iiint_V \left[\frac{\partial^2 T_{ij}}{\partial y_i \partial y_j}\right] \frac{\partial \mathbf{y}}{r} = \frac{\partial^2}{\partial x_i \partial x_j} \iiint_V \frac{[T_{ij}]}{r}\,\mathrm{d}\mathbf{y} + \iint_S n_i \left[\frac{\partial T_{ij}}{\partial y_j}\right] \frac{\mathrm{d}S(\mathbf{y})}{r} + \frac{\partial}{\partial x_i}\iint_S n_j [T_{ij}] \frac{\mathrm{d}S(\mathbf{y})}{r}, \tag{4}$$

where $\mathbf{n} = (n_1, n_2, n_3)$ is the inward normal to S. Eq. (4) has been used explicitly in Curle's derivation (Eq. (2.12) of Curle [2]).

The argument of this section shows clearly that Curle's equation has been derived with a serious mathematical error in the use of the divergence theorem. In fact, Eq. (4), derived by Curle, directly contradicts the divergence theorem, as it means that the flux of a vector through a closed surface Σ is non-zero even if the vector vanishes on Σ.

Note that the above argument properly takes into consideration the retarded time variable and the derivation of Farassat and Myers' equation (20) in Section 4.2 of Ref. [1] does not affect our conclusions.

4. Analysis of the experimental evidence

To support their view that Curle's formula has been successfully verified by experiments, Farassat and Myers make reference to several papers. While trying not to make this response too long, let us consider the experiment described in only one of the papers. Brooks and Humphreys [7] made a comparison of the predictions of FW–H equation with their experimental data. The comparison shows that the discrepancy between the predicted and experimental values of sound pressure is quite large. In fact, the FW–H prediction is generally up to approximately 8 dB lower than the experimental values.

Brooks and Humphreys attributed the discrepancy to some experimental imperfections. However, it can be also explained by the imperfections of the FW–H equation per se. Our analysis of the sound scattering and radiation examples shows that Curle's and FW–H equations in most cases give predictions somewhat lower than the correct values. For example, in the case of a rigid sphere in a variable velocity field [4] the sound amplitude predicted by Curle is three times lower than the correct value, which constitutes the difference of approximately 9.5 dB. This value is close to the maximum discrepancy in the data by Brooks and Humphreys.

It is not claimed here that the explanation of the experimental data provided by Brooks and Humphreys is incorrect. However, their experiment is seemingly too imperfect to be considered as a confirmation of FW–H and Curle equations. More precise experiments are needed for this purpose.

5. The scattering problems and their solutions by AA

5.1. Example 1—scattering of a plane wave by a rigid sphere

Farassat and Myers present two different approaches to the acoustic scattering, both of which are based on Curle–FW–H equations. The first approach implies that the scattered pressure is written as an integral of the total pressure on the boundary, whereas in the second approach the scattered pressure is determined through the incident field on the boundary. A discussion of the validity of these two approaches in application to scattering problems follows. Implications of the two approaches for Curle's methodology are also discussed.

5.1.1. Approach 1: solution through the total pressure and its incorrectness

The first approach is based on Farassat and Myers' equation (5) and uses *the total* values of the pressure and velocity as the source terms. Whereas Eq. (5) represents a formal solution of a scattering problem, the method of finding its specific solution, as demonstrated by Farassat and Myers, is incorrect. Below we show that this method fails to produce the correct prediction in a case where Farassat and Myers claim it to be applicable, namely, in the scattering of a plane acoustic wave by a rigid sphere.

Farassat and Myers' equation (5) is derived from Curle's equation and represents the far field form of a more general equation (Eq. (10) of Farassat [8] with a corrected typing mistake):

$$4\pi P^s(\mathbf{x},t) = \int_S \left(\frac{[P^t]}{r^2} + \frac{1}{c_0 r}\left[\frac{\partial P^t}{\partial \tau}\right] \right) \cos\alpha \, \mathrm{d}S. \tag{5}$$

Here P^s and P^t are the scattered and the total acoustic pressure, respectively, S is the surface of the sphere, c_0 is the speed of sound, $\tau = t - r/c_0$ is the retarded time, $\mathbf{x} = (x_1, x_2, x_3) = (x, \Phi, \Theta)$ is the observation point, $\mathbf{y} = (y_1, y_2, y_3) = (y, \phi, \theta)$ is the source point, $r = |\mathbf{x} - \mathbf{y}|$, and α is the angle between the vectors $\mathbf{x}$ and $\mathbf{y}$ determined by the formula

$$\cos\alpha = \cos\theta\cos\Theta + \sin\theta\sin\Theta\cos(\Phi - \phi). \tag{6}$$

Farassat [8] showed that Eq. (5) above correctly predicts the scattered sound in the far field. Below, however, we demonstrate that Eq. (5) fails to give the correct predictions in the *near field*.

Consider a rigid sphere of radius, R_0, irradiated by a plane acoustic wave of amplitude, P_0, and frequency, ω. If a harmonic temporal dependence $e^{-i\omega t}$ is assumed, Eq. (5) can be reduced to the following equation:

$$4\pi P^s(\mathbf{x}) = \int_S \left(P^t(\mathbf{y}) e^{ikr} \left(\frac{1}{r^2} - \frac{ik}{r} \right) \right) \cos\alpha \, dS(\mathbf{y}). \tag{7}$$

The following assumptions are made. First, the sphere is assumed to be acoustically small:

$$kR \ll 1. \tag{8}$$

Second, the observation point is close to the sphere (near field condition):

$$kx \ll 1 \tag{9}$$

and third, for simpler calculations, it is still possible to assume that the radius of the sphere is much less than the distance from the sphere:

$$R \ll x. \tag{10}$$

Condition (9) leads to the following simplified form of Eq. (7):

$$4\pi P^s(\mathbf{x}) = \int_S \left(P^t(\mathbf{y}) \frac{e^{ikr}}{r^2} \right) \cos\alpha \, dS(\mathbf{y}). \tag{11}$$

The total pressure field on the surface of the sphere has the following form (Farassat [8]):

$$P^s(r,\theta) = P_0(1 + \tfrac{3}{2} ikR_0 \cos\theta). \tag{12}$$

Eqs. (8)–(10) lead to the following approximated equation:

$$\frac{e^{ikr}}{r^2} \approx \frac{e^{ikx}}{x^2} \left(1 + 2\frac{R_0}{x} \cos\alpha - ikR_0 \cos\alpha \right). \tag{13}$$

Substituting Eqs. (12) and (13) into Eq. (11), one obtains

$$4\pi P^s(x,\Theta) = P_0 \frac{e^{ikx}}{x^2} R_0^2 \int_0^{\pi} \int_0^{2\pi} \left(1 + \frac{3}{2} ikR_0 \cos\theta \right) \left(1 + 2\frac{R_0}{x} \cos\alpha - ikR_0 \cos\alpha \right) \cos\alpha \sin\theta \, d\theta \, d\phi. \tag{14}$$

Integration of Eq. (14) gives Curle's prediction of the scattered sound amplitude in the near field based on Eq. (5) as

$$P_{sc}^{\text{curle}}(x,\Theta) = P_0 e^{ikx} \left(\frac{2}{3} \left(\frac{R}{x} \right)^3 + \frac{1}{2} \left(\frac{R}{x} \right)^2 ikR \cos\Theta \right). \tag{15}$$

On the other hand, the exact equation for the pressure field scattered by a rigid sphere, if the incident field is a plane wave, is known [9]:

$$P_{sc}^{\text{pierce}}(x,\Theta) = -\frac{k^2 P_0}{4\pi} \frac{e^{ikx}}{x} \left(\frac{4}{3} \pi R^3 \right) \left(1 - \frac{3}{2} \left(1 + \frac{i}{kx} \right) \cos\Theta \right). \tag{16}$$

With the use of conditions (8)–(10), one can transform Eq. (16) as follows:

$$P_{sc}^{\text{pierce}}(x,\Theta) = P_0 e^{ikx} \left(-\frac{1}{3} \frac{R}{x} (kR)^2 + \frac{1}{2} \left(\frac{R}{x} \right)^2 ikR \cos\Theta \right). \tag{17}$$

It can be seen clearly that Eqs. (15) and (17) differ in their first, monopole, term. This difference leads to the following significant conclusion. Despite Farassat and Myers' claim that their equation (5) of Ref. [1] is valid for the scattering of a plane wave by a sphere, this equation in its complete form (Eqs. (5) and (7) above or Eq. (10) of Farassat [8]) *is not valid in the near field of the sphere*.

5.1.2. *Approach 2: solution through the scattered pressure and its equivalence to Zinoviev and Bies' results*

The second approach, where the scattered pressure is determined through the incident field on the boundary (Eq. (4) of Farassat and Myers), represents the correct method of solution of scattering problems.

However, it can be easily shown that Eq. (4) of Farassat and Myers coincides with Eqs. (44)–(46) of Zinoviev and Bies [4], which we used to calculate the scattered sound amplitude in our scattering examples.

5.1.3. Approaches 1 and 2 and Curle's method

One may ask, which of the two approaches represents Curle's methodology. The answer to this question is provided by Farassat and Myers themselves. After mentioning the possibility to derive an integral equation from the FW–H equation, these authors state that "it is the present authors' opinion that such an approach should not be considered as an application of the AA".

The basis of the Curle–FW–H methodology is stated, for example, in Farassat [10]. In this reference, it is clearly demonstrated that in the FW–H equation the source terms are *not the pressure and the velocity of the scattered/radiated wave*, but *the total pressure and velocity* on the boundary. This statement confirms that, according to Farassat and Myers' understanding of Curle–FW–H methodology, the total pressure on the surface is measured or calculated in some way outside the framework of the integral equation and then substituted into FW–H equation. Unfortunately, this approach violates one of the basic principles of solving boundary value problems, which states that both the total field and its derivative cannot be specified on a surface simultaneously. If one of these values is specified (in Curle's formula the total velocity is zero), the other should be determined from an integral equation (see Section 4.5 of Zinoviev and Bies [5]).

This section provides another proof of the presence of fundamental flaws in the theory by Curle–Ffowcs Williams and Hawkings.

5.2. Example 2—a spherical wave converging on a rigid sphere

We disagree with Farassat and Myers' statement that their equation (5) is not applicable to this problem. We provide the following straightforward argument in support of this view. Farassat and Myers emphasize in their Section 3.1 that "this equation is valid only for scattering by a closed rigid surface of the field incident from a point source exterior to the rigid surface". Since a converging wave can be represented as the field generated by a layer of point sources located on a sphere with a large radius, then, due to the linearity of Farassat and Myers' equation (5), that equation should also be valid for a converging spherical wave.

5.3. Example 3—sound generation by a sphere in a variable velocity field

Farassat and Myers explain the difference between the predictions based on Curle's equation and the expected results by claiming that "the source of the generation of the variable unsteady flow over the sphere has not been modeled in the AA model of Zinoviev and Bies".

Contrary to this claim, the source of the unsteady flow is strictly defined in our previous publication [5]. The variable velocity field is generated by a concentrated hydrodynamic force (point force). Such an acoustic source is well known from acoustic literature [9]. Fig. 1 of Ref. [5] shows a clear picture of the source under consideration.

5.4. Example 4—a rigid sphere embedded in the flow

In contradiction of Farassat and Myers' opinion, we believe that this example is not only a thought experiment, but has a strong basis in the reality. For instance, consider a turbulent fluid flow generated by a nozzle. If a small rigid spherical shell is inserted into such a flow, and if the shell is light enough, it will be chaotically moved by the turbulence so that the difference between its velocity and the velocity of the fluid in its vicinity will be negligible. The immovable observer may be located just apart from the flow in the region where the fluid is stationary. On the one hand, application of the Curle–Ffowcs Williams and Hawkings theory to this case leads to the conclusion that such a shell will radiate sound with the amplitude determined by its velocity with respect to the observer. On the other hand, strict solution of the boundary value problem shows that the shell will not radiate sound at all.

6. Other matters

6.1. The surface divergence

Farassat and Myers claim that the definition of surface divergence, which we used in Zinoviev and Bies [4,5], is ‘‘meaningless and incorrect’’, and provide their own definition of this notion. Whereas their definition may be valid for some specific purposes, the definition which we used in our derivations has a clear meaning. Indeed, the difference between the normal components of a vector field on both sides of a closed surface simply reveals the non-zero divergence of the vector field (i.e. its ‘‘sources’’) on the boundary. Obviously, such sources must be taken into account when applying the divergence theorem to the volume bounded by the surface.

7. Derivation of Curle's equation from FW–H equation

Farassat and Myers [1] stated that the ‘‘Curle formula is obtained trivially from the FW–H equation, thus proving its validity’’. Although re-consideration of the derivation by Ffowcs Williams and Hawkings [3] is the subject of another paper which is being prepared for publication, we consider it necessary to provide a brief argument on this issue here.

Ffowcs Williams and Hawkings consider general mass and momentum conservation laws for a fluid with a surface of discontinuity. Then, they assume that the fluid on one side of this surface is a rigid body, and, to obtain their equation, substitute some boundary conditions into the conservation laws. The boundary conditions, which they use, allow the discontinuity of the mass flow vector and the stress tensor across the boundary.

It is trivial that on the boundary between two different fluids as well as between a fluid and an elastic object the mass flow vector and the stress tensor must be continuous. For an impenetrable boundary the continuity of the mass flow vector means the obvious requirement for the normal component of the velocity vector to be continuous. The condition of continuity of the stress tensor arises from the apparent fact that the boundary itself has no mass. Both conditions are well known in acoustics and used wherever the interaction between an elastic object and a fluid is considered.

Obviously, a rigid body can be regarded as a limiting case of an elastic body (with large velocities of elastic waves) or a fluid (with large density). Consequently, the same conditions of continuity should be satisfied on a rigid boundary.

The simple argument above leads to an important conclusion that the boundary conditions used in the derivation by Ffowcs Williams and Hawkings [3] are non-physical and incorrect.

8. Conclusions

We would like to emphasize the following major points which we have demonstrated in this response.

First, Curle's formula has been derived with a serious error in the use of the divergence theorem. According to Curle's derivation, an integral of the divergence of a vector field over a volume is non-zero even when the vector field vanishes on the boundary enclosing the volume.

Second, Farassat and Myers' equation (20), which has been derived by taking explicit account of the retarded time variable, is equivalent to equations utilized by us. Therefore, Farassat and Myers' calculations do not affect our argument and conclusions.

Third, Curle's equation fails to predict the amplitude of the scattered sound in the near field for a case where Farassat and Myers claim that it should give correct predictions, specifically, for a plane wave scattered by a rigid sphere.

Fourth, the method, based on the Curle–Ffowcs Williams and Hawkings formulation, violates a fundamental principle of solving boundary value problems; namely, the necessity to determine the value of a variable on a boundary from an integral equation, if the normal derivative of this variable is already determined.

Fifth, the boundary conditions between a fluid and a rigid object used in the derivation of the Ffowcs Williams and Hawkings equation are non-physical, since they violate the obvious requirement for the normal velocity and the pressure to be continuous on the boundary between two media.

We also provided rebuttal to other points made by Farassat and Myers.

As a final point, we are disappointed by Farassat and Myers' decision not to participate in further discussions.

References

[1] F. Farassat, M.K. Myers, Further comments on the paper by Zinoviev and Bies, "On acoustic radiation by a rigid object in a fluid flow", *Journal of Sound and Vibration* 2006, this issue, doi:10.1016/j.jsv.2005.07.045.
[2] N. Curle, The influence of solid boundaries upon aerodynamic sound, *Proceedings of Royal Society* A 231 (1955) 505–514.
[3] J.E. Ffowcs Williams, D.L. Hawkings, Sound generation by turbulence and surfaces in arbitrary motion, *Philosophical Transactions of Royal Society* A 264 (1969) 321–342.
[4] A. Zinoviev, D.A. Bies, On acoustic radiation by a rigid object in a fluid flow, *Journal of Sound and Vibration* 269 (2004) 535–548.
[5] A. Zinoviev, D. Bies, Authors reply to: F. Farassat, Comments on the paper by Zinoviev and Bies "On acoustic radiation by a rigid object in a fluid flow", *Journal of Sound and Vibration* 281 (2005) 1224–1237.
[6] M.J. Lighthill, On sound generated aerodynamically, *Proceedings of Royal Society* A 211 (1952) 564–586.
[7] T.F. Brooks, W.M. Humphreys Jr., Flap edge aeroacoustic measurements and predictions, *Journal of Sound and Vibration* 261 (2003) 31–74.
[8] F. Farassat, Comments on the paper by Zinoviev and Bies "On acoustic radiation by a rigid object in a fluid flow", *Journal of Sound and Vibration* 281 (2005) 1217–1223.
[9] A.D. Pierce, *Acoustics: An Introduction to Its Physical Principles and Applications*, Acoustical Society of America, New York, 1989.
[10] F. Farassat, Acoustic radiation from rotating blades—the Kirchhoff method in aeroacoustics, *Journal of Sound and Vibration* 239 (2001) 785–800.